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Abstract:
By statistical research valuable information about properties of the game of Go can be achieved. Tournament results concerning points of victory reveal a scattering parameter to be related with strength of players. The position an distance of the absolute best game may be estimated to be 1 to 2 Handicap-Stones better than now a 9-Dan profi.-player. It has been shown that the distribution-curves of one Handicap-Stone has been explored to be rather the value of 14-18 points.

Introduction
Though it is possible today to have far-reaching and important theorems about the game-theory, it is rather difficult to control the theory by direct measurements, i.e., practical verification. Just with Go however, this appears to be quite easily possible.
Hereby we want to discuss some aspects of such measurements and statistics.

I. A Method for statistical measurements:

It has been known for a long time that the strength of players significantly decides upon the result of the game, that is to say the score. But it is as well known that even with equally strong players the result of the game (score) cannot be predicted exactly as there is a certain random scattering observed around the average result to be expected. This scattering is very big with weak players, with the better Players it is smaller, and the better the playing strength the smaller will be the scattering. So we can make use of this effect to determine the average strength of a class of nearly equa- players independantly and as an „objective Method“.

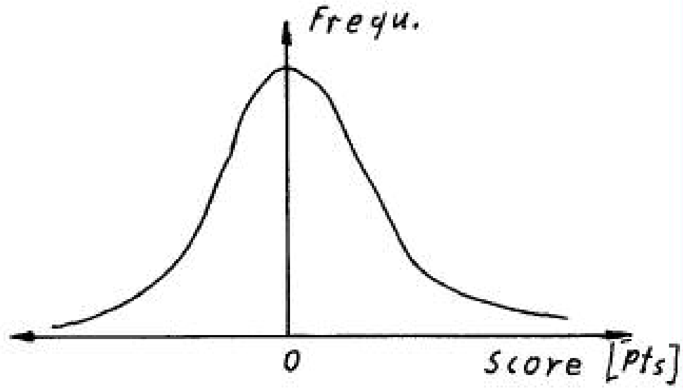

FIG:1. DISTRIBUTION-COURVE

For this purpose we must evaluate a rather big number of games especially with respects to the score. The very best way to do this is to take tournament results of rather many- let´s say 10 (-nearly-) equally strong players, as shown by the table - Fig.2.) of a Europ.-Champ. Tournament 1975 in Krems.
By plotting all such results here,46 games, into a graph with probabilities,we get a distribution that may be approximated by a straight line. With respects to the normal distribution of random scattering values we define a scattering-parameter that includes 203 of all the games, to be the standard-scattering-parameter „r“.

◩:Black ◸:White ↓Results as played

Nr	Name	KL.	1	2	3	4	5	6	7	8	9	10	11	12
1	Matern	9,0	≡	+a	+wo	—	+23,5	+a	+a	+12,5	—	+9,5	+a	—
2	Wimmer	10,2	-9	≡	+wo	+8,5	+a	+a	+4,5	+a	—	—	—	+a
3	Katscher	10,5	+8	-14	≡	-a	+13,5	+7,5	—	-0,5	—	—	+11,5	+55,5
4	Wittschek	11.0	—	+4,5	+18	≡	-a	-a	+0,5	—	+33,5	—	+17,5	+a
5	Merissert	11.8	+7,5	-7	-6,5	+19	≡	+a	—	+2,5	+10,5	+a	—	—
6	Rehm	12.1	0	-6	+1,5	+20	-14	≡	—	—	+a	+2,5	—	+8,5
7	De Vries	12.1	0	+4,5	—	+4,5	—	—	≡	+8,5	+3,5	+9,5	-a	+7,5
8	Novak	12.6	-5,5	-3	+11,5	—	+1,5	—	-5,5	≡	+a	-10,5	+4,5	—
9	Greb	12.8	—	—	—	-24,5	-5,5	-11	+0,5	-14	≡	+a	+35,5	-5,5
10	Bates	13.4	+12	—	—	—	-7	-2,5	-1,5	+14,5	-12	≡	+4,5	-a
11	Kitsos	13.5	-7	—	+3,5	-3,5	—	—	+22	-9,5	-32,5	-4,5	≡	+a
12	Sudhoff	14.4	—	+6	-35,5	+2	—	+3,5	+4,5	—	+13,5	+20	-13	≡

↑Results after corrections applied.
(Resignation 15 Points).

Fig 2. European Campionship 1975

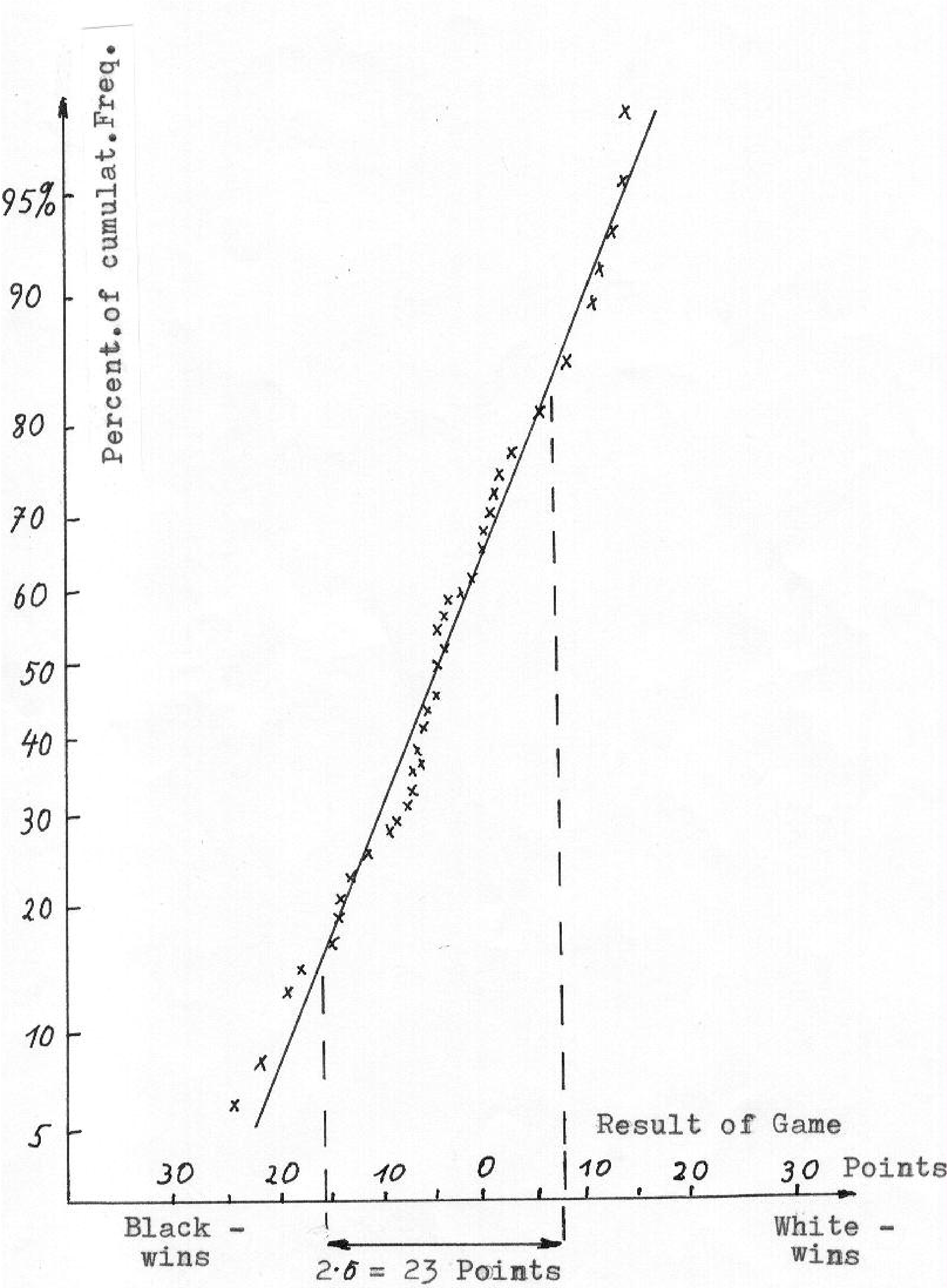

Fig 3. Evaluation of scattering parameter with probability scaling

Within this range of „r“, here to be +11,5 points of margin of the score,we observe to have 31 games. Only 15 games,i.e. 7 times black wins and 8 times white wins by a bigger score.

Correcting considerations and adjusting:
As a matter of fact it is not easy to evaluate the statistical data due to some systematical effects.

a) Correcting for players real strength.
   Very often the playing strength of participants is not really exactly the same. So their chances are not even and winning and loosing-margins may be encreased. Therefore it is necessary to correct their winning and losing points in respects to actual playing strength as shown out by the tournament itself.(According to old fashion here:1stone is still counted to be worth = 10 points.) For example: a 4-Dan(W) wins against a 3-Dan(Bl) with +12 points on the board. First we have to consider the 5 points for comi that Black has to give White for the first move. But as White plays stronger by one Dan = 1 stone he,in the average, has to win by 10 points better. the victory of White then is to be considered as following +12+5-10 = 7 points as the corrected value.
b) Many players sometimes do not finish the game properly, but they just give up and resign when they consider the fight to be in their opinion hopeless. For statistics crude estimation is still possible here if we assume that the value of resignation is approximately the amount of the scattering-parameter (or the playing strength in the European Classification System).
   For example: A 1-Dan(BL) plays against 1-Dan(W) and he resigns. So we have to count: 1-Dan = 18-Europ.Class. that is 18 points. And there are as well 5 points of comi to count for here. So the victory for White should be regarded 18+5=25 points. -->> By these corrections the results of tournaments can be very much ameliorated from the point of view of statistics. A tournament of 10 participants already gives reliable statistical data. And as can be seen by Fig.3. those statistical data may very well be plotted on a statistical graph for cumulative probabilities.

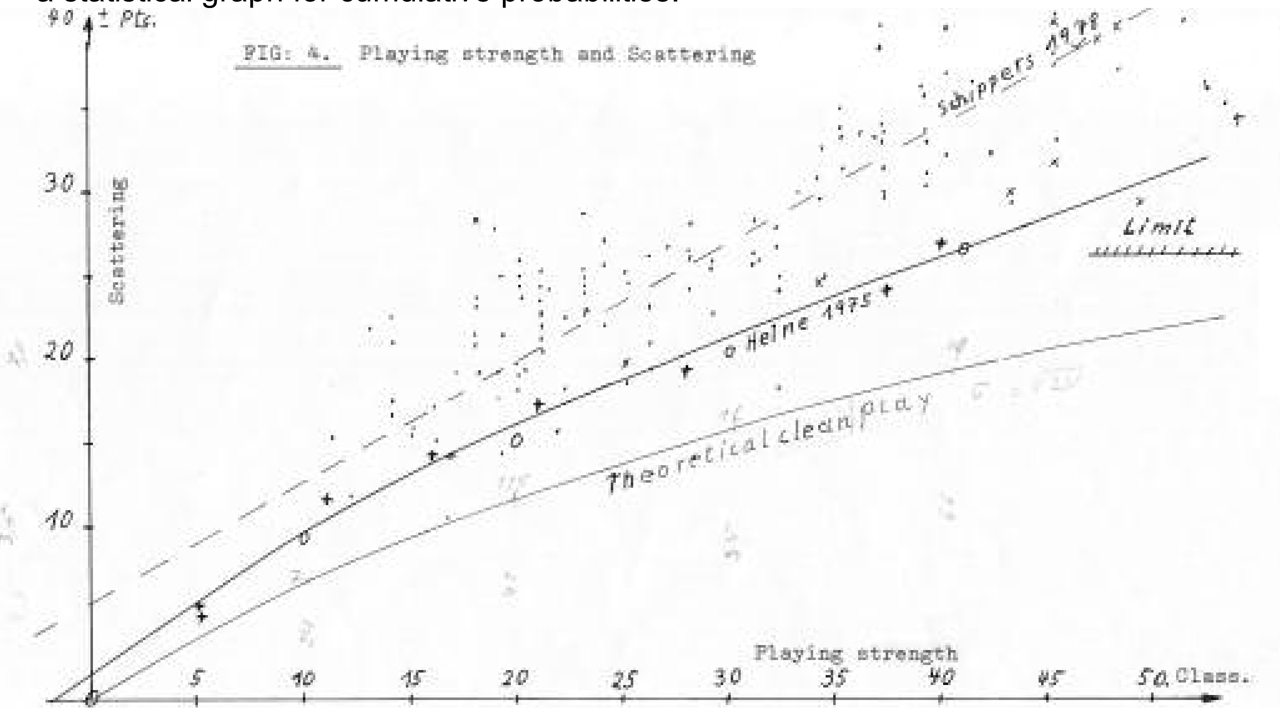

Fig 4. Playing sterngth and scattering

II. Discussion of results:

The amount of scattering and the function of strength: It yields the relationship between the amount of scattering and the strength of players results from the undertaken research with such a considerable precision and reliability, that a further discussion seems already to be possible.(Fig.4) This is the line in the graph (Fig.4) designated as ´Heine-1975. Each x-

point results from the evaluation of one tournament. .... As an example how difficult it is to have reliable data may be seen, if just 2000 games are lighted on a computer of normal club-games (Schippers 1978) which here were plotted as simple.... points. Then the average - - - dotted-line lies some 25% above the standard values. But then, after critical elimination of all handicap-games and proper application of correlation's and corrections,the values as indicated by the „-0-“ points ly again almost precisely on the standard curve.
Though these evaluations give reproducible and precisely the amount of scattering in the relation to the strength of players, there are reasons from the theoretical view to look out for smaller values of scattering as drawn by the lowest curve indicated as „Theoret-clean-play“. Some aspects of this thinking is being argued by the report of W.Kramarczyk or K.Heine and may be derived from the „Bimodal-effect“ This theoretical curve should give a considerable better approximation to $\delta = \sqrt{2N}$ according to theory.
As the curve in Fig.4designated here as ´Heine-1975´ may be considered as sufficiently reliable and proved,we now are able to discuss some outstanding properties that are to be observed by this graph:

- There certainly exists an optimal way of playing without any point losing errors and scattering. This way of optimal play is for our 9-Dan Profi-master-players still 2 Handicap-stones ahead (with +1 stone, precision).
- Studying statistics we see interesting properties so that several effects of the game can better be understood.

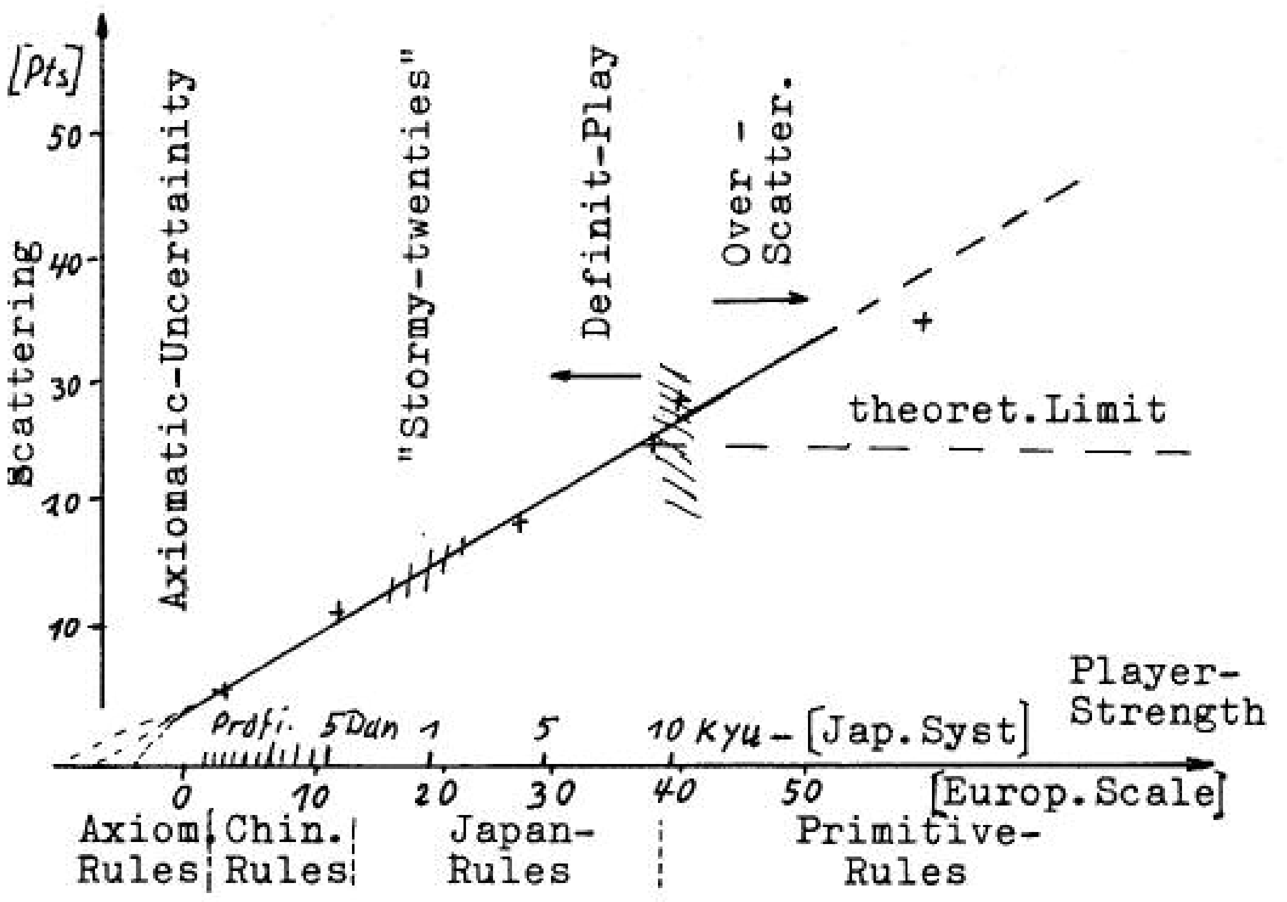

Fig. 5 Areas of different play

- By considering effects along with the curve (Fig.4:Heine-1975) we may recognise several regions of interest that we can already classify today by arguments of the „Theory of Errors“ and looking on some reasoning about“Information“(see Fig.5).

There are some special points (areas) along the scale of player´s strength to note:

A) Around the strength 40-European-scaling, or-10-Kyu, the number of error points starts to become smaller than the entire value of the game + 360 points.

B) Around the strength 20-European-scaling, or-1-Kyu, „the stormy 20ies“, there seems to be an area where there is especially stiff fighting. there are tournaments where there are only resignations with all games.

C) Around the strength of 5-7.Europ.scal, or with the profi-players,starts a complete understanding of perfection on the board. (Already a feeling of The-Perfect-Game)

By this observation we also can understand the oriental way of classifying:

- Total novice to the game;
- Kyu, = i.e. learning the game = apprentice.
- Amateur Mastership;
- Professional Mastership.

There seem to be some reasons for this kind of classifications.
But here it must be pointed out that, from an objective point of view and with respect to mathematics, those classifications only represent human errors and human psychological properties as reflected by the game. the game itself (its mathematical strict rules) is independent of such properties. The kind of errors that are made with the different classifications of strength should as well be looked upon as neutral.

For conclusion we stat that a linear proceeding scaling from top-strongest to weakest players must therefore definitively be advocated in respect to the game of zero - sum with complete information.
There are several scalings of this kind thinkable. As a matter of fact the European-System as such is the only one that takes care of these fundamental relations up to now.

Fig. 6 Probability – distribution as a function of score

III. The Bi-modal-Scattering Distribution:

A more detailed statistical evaluation with more material, which now is already available (with always 100 - 1000 games) reveals that in most cases the experimental curves do not follow a stochastic normal distribution in reality. See Fig.6.
The curves show uniformly two peaks. This seems to apply to all ranges of player´s strength, especially to the strong professional masters.
The games with resignation, whose judging have always been difficult in statistics, play an important part, but not a decicive one. Here we have always assumed that this will be the value of player-strength according to Europeanclassification in points.
But due to the non-standard shape of the curves,which is always observed, it must be pointed out that we should no longer talk about a standard deviation „ δ “ but rather

about a parameter of scattering „r“ in order to avoid confusion. This „r“ should be understood to contain and to stand for 68% of statistic mass.

IV. Scattering and Values :

From an orthodox point of view we may regard as matter of fact the relation $\delta=\sqrt{2N}$ for scattering. (This only accounts for real stochastic procedures and a normal distribution derives from a statistical mass.) By doing so we can conclude from δ to N by writing $N=\frac{\delta^2}{2}$. As this holds true for all strength of playing, we conclude: scattering ranges along the entire functional curve $r=f(strength)$, - here with the meaning $r=\delta=2N$. (See Fig.7) This means that N is the entire number of all possible counting error-points, and finally N is the number of points given by the value of the game.

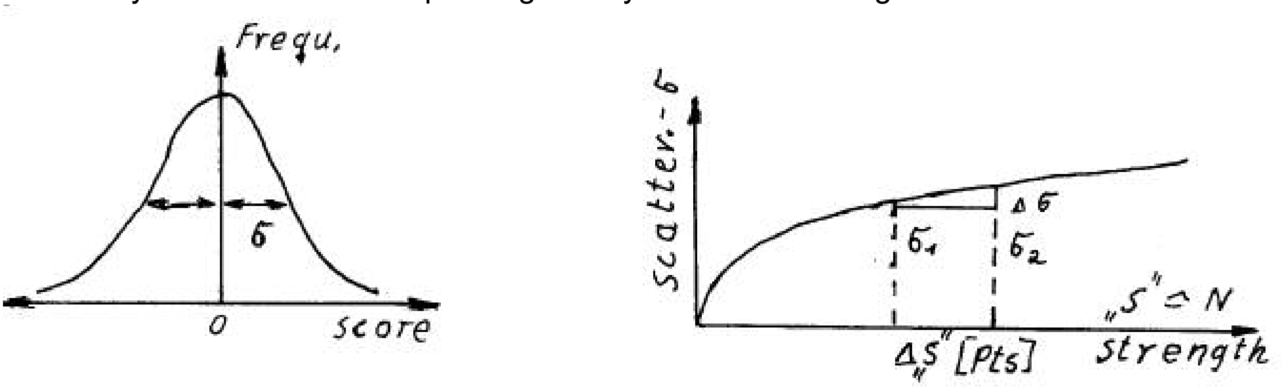

Fig 7. Scattering and number “n” of points

This case, however merely theoretical, seems in the case of the game of Go somehow to be changed as the form of the distribution-curves prove to be bi-modal. Despite this irregularity here as well, there is some relation between the scattering parameter „r“ and the volume of the statistical mass „N“ below the distribution curves. Only here the mathematical relation seems to be more like r and N, so that the function $r=f(N)$ should rather be expected to be a straight line than a parable. And just this is observed.

V. Other Projects for statistics of Games:

a) The value of the first stone amounts to 14 - 18 points. Quite exact measurements exist already now about the value of one, i.e. the first stone. (See Fig. 8) The following studies already do exist:

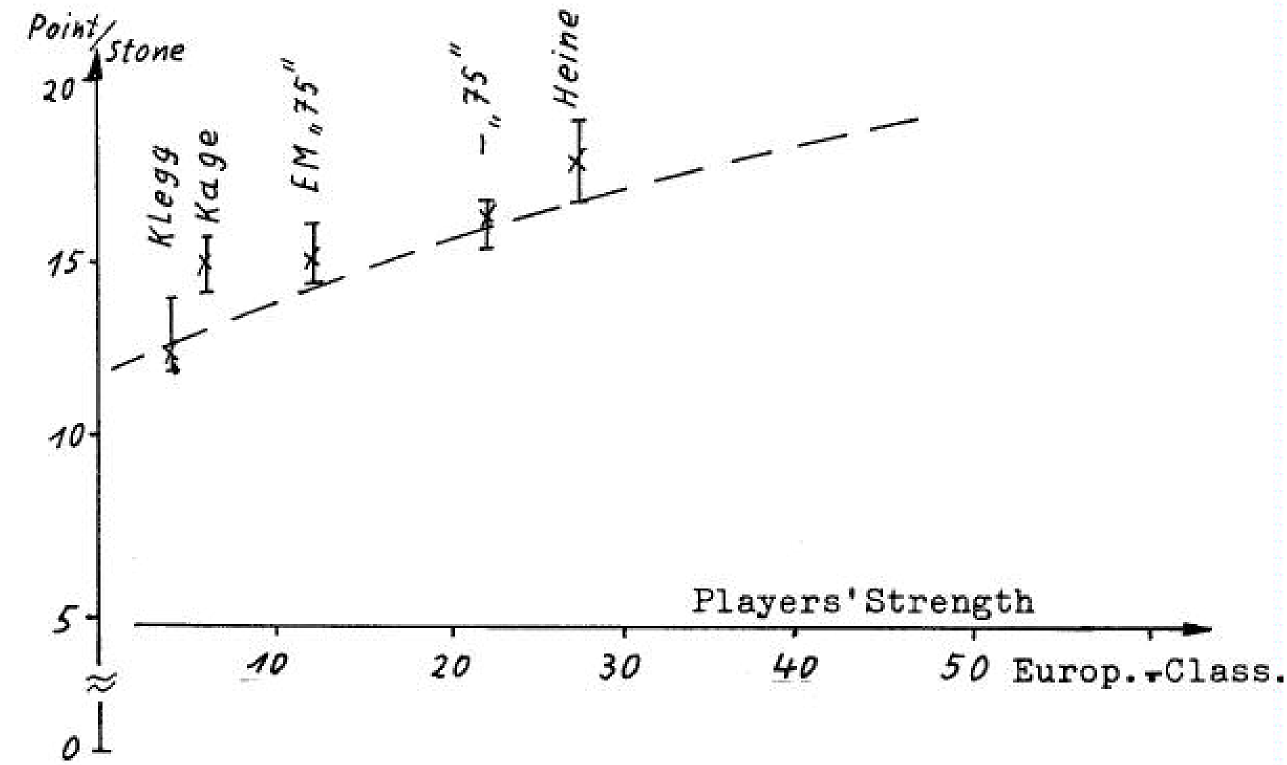

Fig. 8 Value of first stone as a function of player's strength.

b)

	Number	Strength	Precision
Klegg: on Profi-Games	2000	6.-9.Pr.Dan	+0,5 Pt.
Heine: Kageiama	10 x 4	5. Pr.Dan	+0,8 Pt
Heine: EM 1975	40	4. Am.Dan	+0,8 Pt
Heine: Own Test-Games	5 x 5	4. Kyu	+1,0 Pt

The most successful and prospective results are measurements of games that are played by evenly strong players, where alternative by one of them is taking 3 to 5 stones of handicap.

c) At the end of the game there is always an area of 50 - 60 points left, as may be observed with all profi- and amateurgames. It is of high theoretical interest to state that there is apparently no perfect balance between fighting by attacking the enemy and looking after territory. But it seems to be more useful just to play more for security and defensively-safe... >> Why is it like this ? <<

d) An important statement about quality of errors and way of playing may be made by comparing games starting off from the 100. - 140.ties move that are played once between profi-profi-players and once alternatively between profi-amateur players. The results (or,i.e...the difference of results) of each the games may be compared, and the most important thing here is: they can exactly be calculated as they are all endgame positions.

e) It should be very interesting to correlate ways and strategies of games on small boards. This is the case because it is much easier to calculate the basic problems of Go on small boards tnan on big ones. One example for this may be the value of the first move. (see Fig: 9) The value of the first move is quite precisely known for two cases:
5 x 5 board: 1.rst move 25 points
19 x 19 board: 1.rst move 6-7 points.

A Function of $W \sim \frac{1}{n}$ seems to be more likely now! compared to a law by the square.

f) Further Projects for Psychology and Pedagogic:
As Go is quantitative and consists of a highly structured way of thinking it certainly constitutes in an outstanding manner a means for the measurement of the human thinking process. So it is rather surprising how few attempts really have been undertaken in this direction. We therefor like to propose further research:
It will be necessary to make measurements on a still bigger number of learning curves with: young people, adults, old people, male and female.
What is the process like of learning with special intensified type of teaching?
What relations are there between time of thinking and strength of Play?
What is the value of team-work? as for example this arrangement for play:
- 2 - players against one player without consulting,
- 2 - players against one player with consulting of the two,
- many players play against one, same way as above.