**Abstract :**
Looking on Go as a very logic and finite system we count values of the Game namely by points. The maximum amount of victory is ± 360 = 720 points. The entire value of handicap stones may be 50 x 14 = 700 points. The maximum scattering as well indicates ~ 700 pts. The cumulated values per move indicate ~ 700 points too. Estimations for the entire information content of the game give i ≃ 1600 bit as likely theoretical value. Discussing ambiguities with the rules of the game we find much redundance within the game. Concluding considerations lead to the discussion of extreme-principles and how we have to think about the formulation of values.

## I. Introduction.

The logic course of a game between two opponents is given by the mathematical definition of a: Zero-sum game with complete information. This means that with perfectly and clearly defined rules, and a finite number of moves the gain of the game is divided in such a way that the loss of one player will exactly be given to the other player. The game of Go (Wei-Chi ) fits these conditions in rather a perfect way. Therefore thinking about the game should not be restricted to considerations of the best moves but there should be a deeper theoretical penetration of far-reaching importance as well.
One aim and application may be: as to investigate a more efficient a more efficient logical system for Artificial Intelligence. Furthermore there are other areas of interest and science partially only connected to Artificial Intelligence. However here with us this is research about information content within the game, and most important connected to this, considerations about values by points as counted in the game of Go.
All properties of this kind shall be analysed here as far as it seems possible to-day. However it is a pity, this can only be a tiny start as all works of more profound understanding of redundance and cob-web-net are still very poor. Furthermore there does not yet exist any start of algebraic topology of Go and grid-theory.
Practical calculations of information content of special situations and examples are not yet known. Despite these difficulties we should start to analyse this game in respect to its information content now by first looking at values. That means , points at all stages of the game and of the entire counting.

## II. Values

The outstanding property of Go can be stated by the fact that the calculation of victory directly corresponds to the way and quality how the player has performed his task. The number of points we call here just „value“, is remarkably reflected by several observations:

### IIa.) Maximum amount in victory.

The highest margin that practically may be realizes amounts to ± 360 points. That is, the strength of playing not only shows itself by the victory of one player, but rather more precisely by the high of victory that is measured by points ( score ).
The full amount of 360 points however is only realized if unequal opponents of vey different skill and strength play against each other. If opponents of nearly the same strength play, the result of the game will be only some few points. Only by a large number of games such results of player´s strength can be cleared from statistical fluctuations.
Thus taking average, by forming mean values of playing strength, we may even think of fractions of a point for differences in playing strength.
Considering extreme situations of game playing, the maximum of 361 points even can be exeeded. But this is a purely theoretical case of „negative playing“, - as we may argue that on the one hand a very week player puts more and more stones on the board and on the other hand his opponent keeps passing. By such a procedure it is possible to achieve a result of more than 361 points. Strictly speaking we can only state that such games are extremely unlykely.

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**II.b.)** **Value by handycap stones**

There is the possibility in Go to give handicap stones at the beginning of the game by just making pass-moves. It may be noted as a very remarkable fact that for all playing strength the result of the game will be affected by 12 - 18 points per stone. By more handicap stones bigger differences in results will be achieved. By this means it is possible to make up for differences in playing strength in a very smooth way.

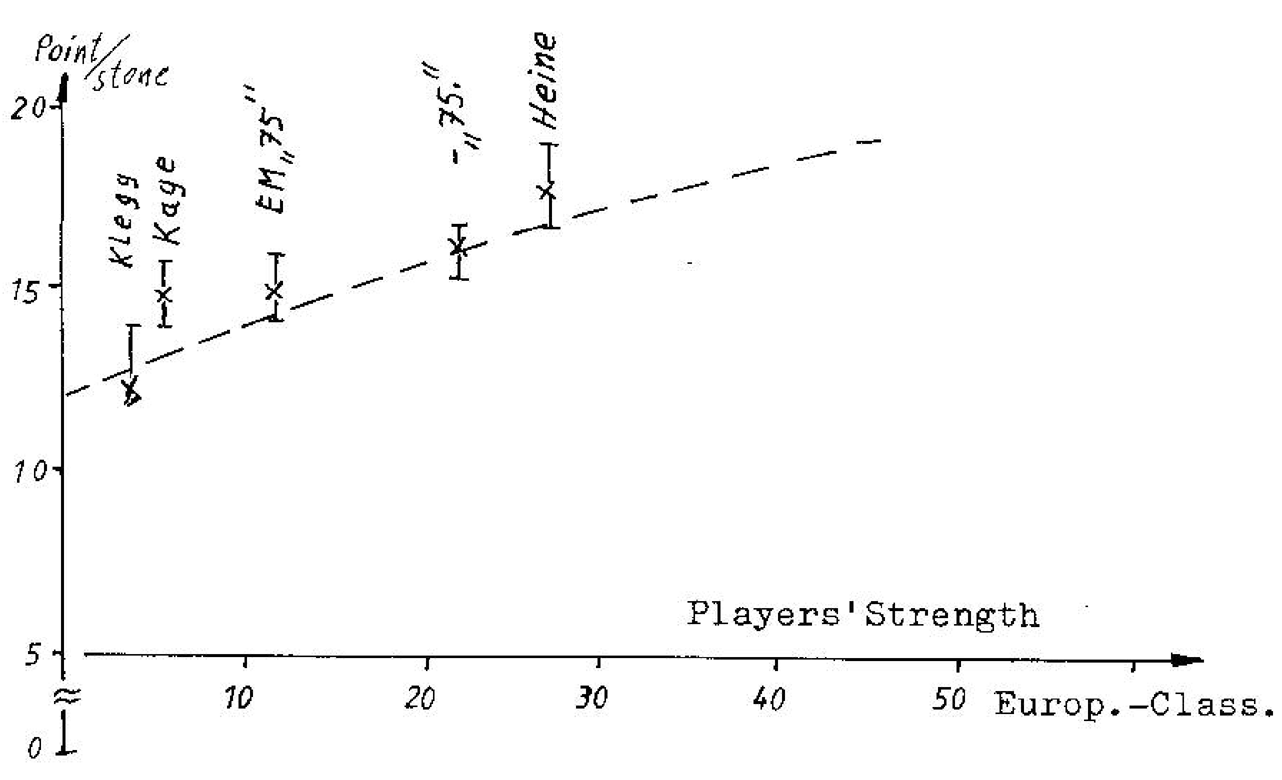


FIG: 1. Value of first stone, - as a function of players' strength. -

It seems to be a wide-spread error that more handicap stones must support each other and so increase4 in value of points ( 9 handicap ~ 140 points ).

There are other important arguments that may be considered in just the opposite direction:

- One stone certainly is more worth ( a little ) than 12 points, rather 14 - 18 points. (See report Heine: „Statistical Research" ). The higher gain can be explained by this.
- The high concentration of many handicap stones should rather effect the opposite, as each stone looses a part of his distance-influence. Thus by the effect of over-concentration the value of a handicap stone should properly diminuish to a certain degree!

Taking into account these considerations we put 14 points for one handicap stone. The entire range from strongest to weakest player may be considered with 50 handicaps in difference of strength. Thereby the value of all handicaps covering knowledge of Go is 50 x 14 = 700 points. This compares very well to the number of ± 360 = 720 points as discussed above.

**II.c.)** **Scattering**

In every normal game there will be always some fluctuations of results due to different evaluations of situations. After many games there will be the average of 50% wins and defeats for both players, evenly strong players. But as can be seen by Fig.2 there is a certain amount of fluctuations i.e. the scattering probability let´s say of ± 25 points of win and loss.
Now the most important fact to be observed can be seen by the correlation of player´s strength and the scattering parameter ( Fig.2 ), as towards greater strength there is a steadily smaller scattering that tends to become Zero. So we get confirmation that there

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exists one strongest way of playing that, looking at the result of the game, does no longer imply that points were won by faulty playing of the opponent.

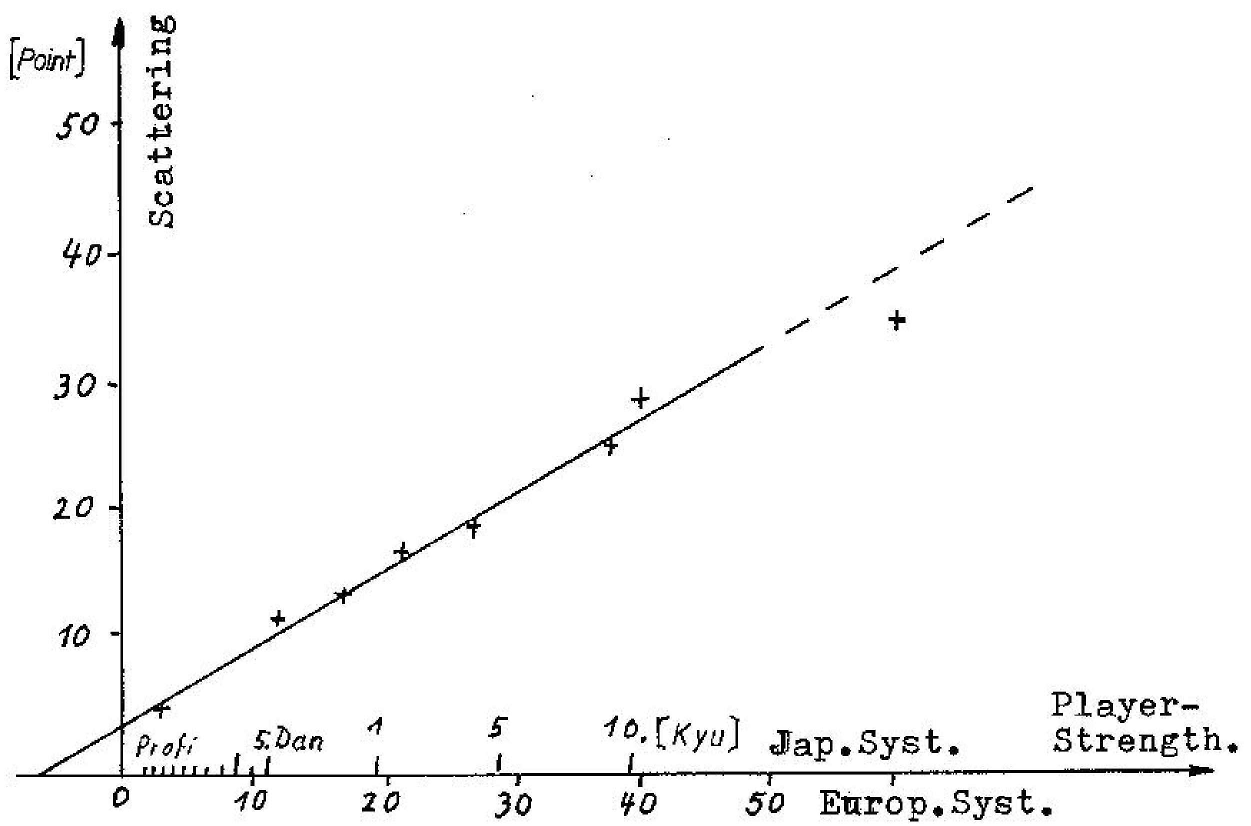


FIG: 2. The function: Scattering ???

A way of playing that deviates from such optimum contains points by error. It need not necessarily follow that there is only one single procedure of play that is optimum, but there is might many ( redundant ) similar ways of playing.

Looking at values by the amount of scattering we relate our experience with properties of a theoretical normal distribution of errors where $\delta=\sqrt{2n}$ should be correct; s,here being a theoretical scattering parameter referring to results of games. If there are not more than $\pm 360$ points to win in the entire game, thus there should not be more scattering than: $\delta=\sqrt{2*360}\simeq 28$ points of maximal scattering. Looking at the evaluation of measured values for weak players along the „theoretical curve“ (see Fig.4 in report „Statistical Research“; K. Heine ) this statement really seems to be endorsed.

**II. d)** __**Values per move**__

With the properties of the game described above there exist still some special facts that merit cansideration: In the beginning, when the board is empty one stone is equivalent to about ~ 14 points. At the end of the game its value is only some few points and finally only 1-point. Neutral moves „damé“ are worth 0-points, as they cannot give any contribution to value. This observation has been put down in Fig. 3 for two different strength. It should be stated that here the straight lines do not simply connect the values of the first move with the last move ( ca: 100 - 200.th move ), but there are more forcing arguments in the beginning to 20 - 30th moves for discussing their values. It is clear that by putting gradually more and more stones on the board their respective single value is gradually decreasing, i.e. is getting < 14 and less points.

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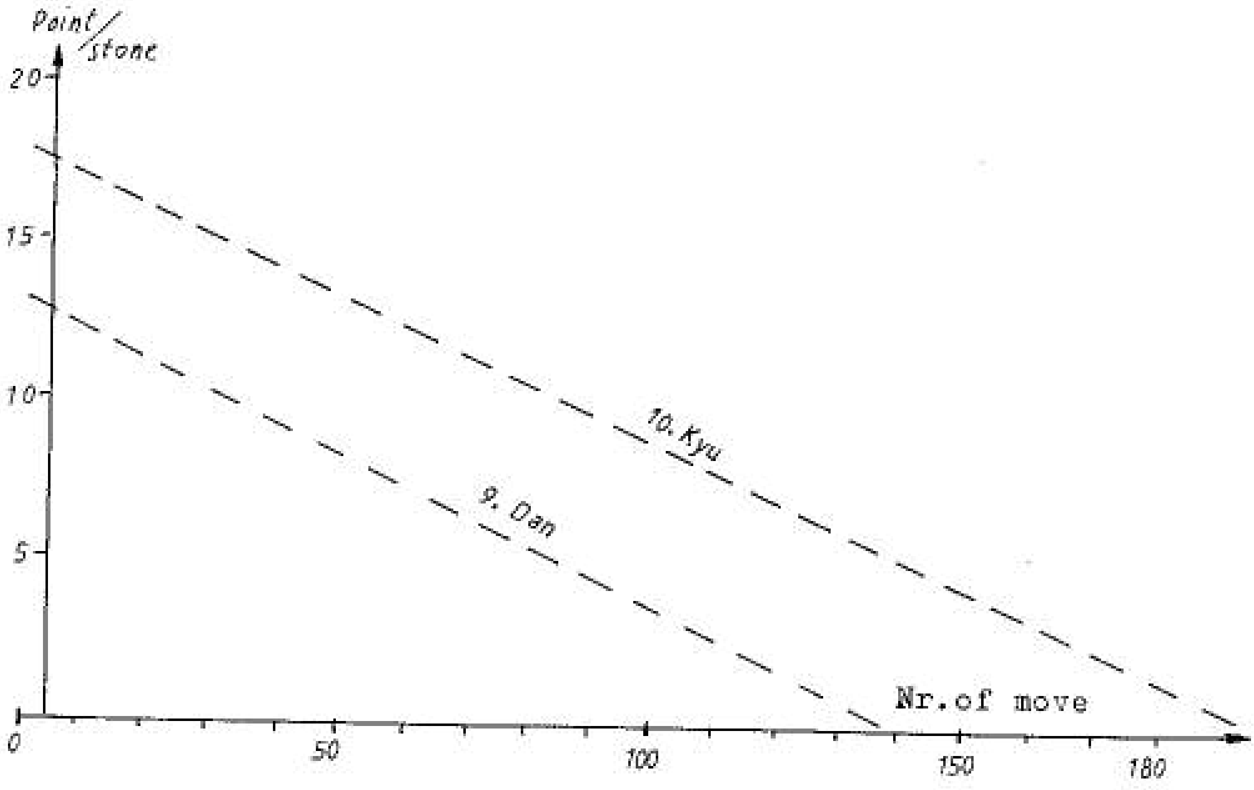


FIG: 3. Value of moves in course of game.

An analogous argument holds for the endgame, where already many moves before the final play, the moves can be counted by decreasing values of points. Between both sides of the curve there does furthermore exist not only the experience but the definite and firm declaration of many strong profi-masters, that continuous optimising of all the moves will be best play. This just means a linear degression of values of the moves (points/stone ).
Only because the theoretically best moves mostly are unknown the value of real moves sometimes varies rather drastically.
If we now add the values of the first to the 100th move we again get t ~ 700 points as total value of the game. Furthermore taking 700 points the entire value of the game and a linear degression from the first move ( of 14-points ) to the last move we can calculate the number „ $n_x$ “ of optimal moves as $700=\frac{1}{2}*n_x*14$ ; with $n_x=100$ moves.
Games of beginners that do contain more moves and where the first stone has more value than 14-points i.e. 18-points appearently seem to sum up a larger total value of the game. But just as with the scattering function and the bi-modal distribution effect ( see the report Heine: „Statistical Research...“ ) we can here assume redundance to be the cause of epparently more value.
Summarizing our considerations of values we see:
a) the amount of maximum victory: $\pm 360=700$ points;
b) entire value by handicap stones $50*14=700$ points;
c) scattering $\delta=\sqrt{2n}\,;\,\sqrt{2*n}=700$ points;
d) values per move $\frac{100*12}{2}=700$ points.
The many properties of Go indicate a total value of the game certainly to be more than 600 and less than 900 points. Most likely to be 720 points. this value strongly correlates with the performance in the game. This means that information about the game is represented, if not directly , but by the value in points.
Mathematically we may stress it even further by stating: Information is additive, so are the

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„info-points“ of a move. Information describes a probability on a generalized look-ahead procedure, exactly this is performed by a move.

**III. Estimation for Information content**

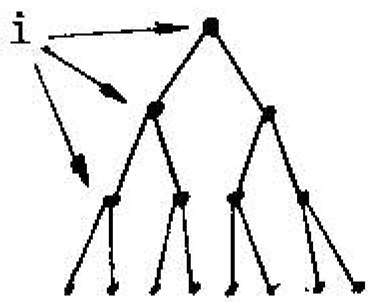
At the moment it will not yet be possible to have exact calculations for the informaton content in Go for each single move and all possible positions. To be able to do this too many facts, too many premisses are still missing like: the exact value of single moves, better understanding of the rules, way of counting. Thus we still must rely rather on estimations that still have to rely on very crude combinatoric models and we still have to compare results with statistical experiments.

**III. a) Combinatoric models for information content. *)**

According to a model of a look-ahead procedure on decisions making, („look-ahead tree“): the information „ i “ describes in an exact manner how to reach N-states of a system:

$N=2^{i}; \; i={}^{2}\log N = Ld\,361!$

$i=Ld\,10^{765}=3{,}3*765\simeq 2540\,(bit)$



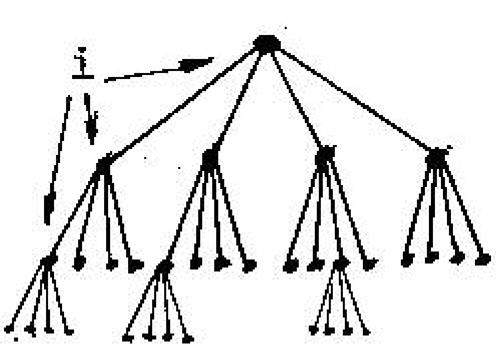

This we may call the information for describing the entire system.

In Go a board having $n*n$ lines, there is $n^2=19^2=361$ . The number of states being $N=n!=361!=10^{765}$ this is to come, as we may consider all possibilities of making different sequences and moves on the board; even independently from colour and not respectively to their real efficiency within the game. We may call this a „combinatoric primitive model“.

Another somewhat more convincing model on a static basis consists of considering complete positions where all the points may be occupied by Black, White or Zero. Thus each point may be described by 3 states, and then the entire composition of all states on the entire board takes the number $N=3^{361}=10^{171}$ of all possibilities. This may be called a „static primitive model“ $(i=3{,}3*171\approx 565\,bit)$ .

**III.b) Considerations for better calculation:**

1. Black and white stones (moves) may kill each other until a repetition of position on the board forbids playing of special moves.
2. The game only makes sense if there are continuous chains on the board. The stones cannot be distributed nor put on the board by any random scheme. The same argument is true, too, for clean „area“ i.e. the connection of empty points.
3. By putting a stone on the board as a matter of fact this makes not merely a decision of „Yes“ or „No“ but in reality there is at least a decision on the 4-liberties giving a 4-fold logic. then we should better write a model of information: $i={}^{4}\log N$ .
4. By this consideration the density of states i.e. number of possible moves is very much increased. However this multitude of states is considerably reduced by netorklike interjunktions. In terms of the game these are inversions of movesquences and sveral symetries of different kind that all end up by the same value in countingof results




So we might conclude that „redundance“ takes an important part in the current game. Possibly we even might look at the vast magnitude of moves apparently equal with respect to the final counting , - and to be properly the main part of the gain, whereas the true differ-

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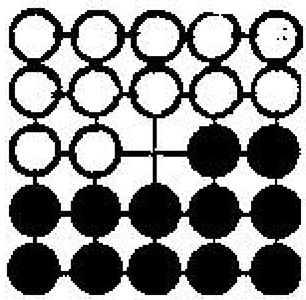
ences in counting points are only the smaller part.
Furthermore here we have to state that going from counting points to neutral points must be considered as continuous. As for example a combination of 7 moves may result in a gain of one point, then we must count by fractions of points as $\frac{1}{7}$ point/move.

### III.c) <u>Example-calculations:</u>

Combinatorical computations for more detailed analysis of real Go positions are always very fatiguing. Two model-calculations are shown here:

1. The biggest number of possible moves:



There is the rather silly method for making moves, we may call it „Anti-Go" or „Idiotic-Go". According to the rules of Go the board (here for example on 5 x 5 board ) may be filled with stones in a rather stupid way by putting one stone next to the other from each side starting till Bl.25 throws all white stones off the board. The game then proceeds in the same way till a constellation will be reached where W. can beat all the Bl. stones. The procedure will be continued so far till all points have been once the last free point for killing. Only after this a repetition will be possible and thereby forbidden,

The „game" is finished. This procedure has the advantage of easy calculation: For a 5x5 board is $m=5^2=25$ ; it is, because the last killing-point don´t count $\left(\frac{m}{2}\right)!*\left(\frac{m}{2}\right)!=12!^2$ .Another killing will take place at $11!^2$ and at $10!^2$ ... etc. then finally we can put for all possibilities the amount N for all possible moves: $N=12!^2*11!^2\ldots 2!^2*1\approx\left(1{,}4*10^{44}\right)^2=N\,2*10^{88}$ .

For the big board of 19 x 19 we now can calculate in a similar with some mathematics:

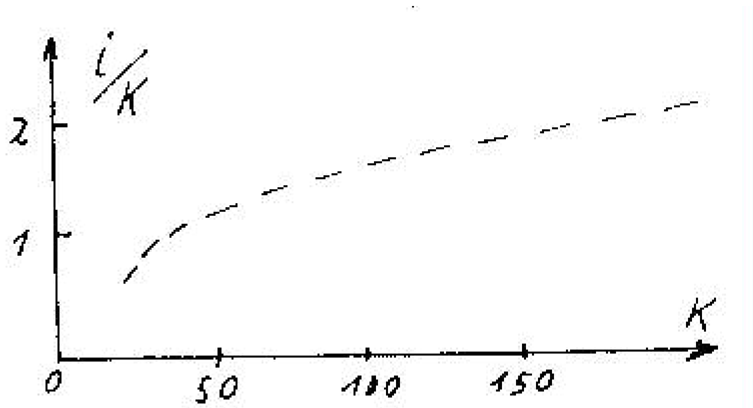
$$N=\left[1!\,2!\,3!\ldots 179!\,180!\right]^2$$

$$\text{or}=\left[\prod_{K=1}^{180}10^{i}\right]^2 \qquad i=f(K)=\frac{K^{1,3}}{2{,}93}$$

$$N=10^{2*\sum_{K=1}^{180}K*\left(\frac{K}{25}\right)^{0,32}}$$

$$N=10^{\frac{2}{2,93}*\sum_{K=1}^{180}K^{1,3}}\approx 10^{\frac{2}{2,93}*\frac{180^{1,3}}{2,3}}$$

$$N=10^{\frac{0,68}{2,3}180^{2,3}}=10^{4100}$$




As this case, here considered, still contains many symmetrical moves, this is certainly not yet the biggest number of all possible moves.

According to Gottfried Schippers the real absolute biggest number of all moves for Go are $N=(3361)^{3361}$ , as we have to consider not only all the static positions, but as well as consecutive possibilities after each capturing.

$$N=10^{170}*10^{170}\approx 10^{10^{172}}$$

By this puzzling with big numbers we do understand directly that the number of all possible moves is much bigger than $10^{760}$ and thereby the content of information should be i >> 2000. But as there are not more than 360 Bl. or W. points to gain certainly there are no more than 720 points. of information possible. Thereby again we are forced to admit that there is very much redundance in the game of Go. With all these considerations we again recognice a difficulty: which of the moves can properly be called to really belong to the game?

2. Restricting Aspects:
   There are quite a few aspects that indicate a drastic reduction of possible moves for a real Go-game intelligently played. In the first place it will be necessary to recognise that in the game each of the players has to have at least one group of stones that consits of coherent rows of stones. this means,that of the 360 points Bl. and W. each has about 178 stones and each has 2 empty points for his own. The number of possibilities the is computed in the following way:
   $$N = \frac{361!}{4! * 187!} \approx \frac{10^{765}}{10^{320}} = 10^{445}$$
   Furthermore we should consider that in a real game there are generally 2 x 60 empty points and about 40 points of neutral „damé"-points between the chains, then: $N = \frac{361!}{160!} \simeq 10^{482}$ and thereby: $i = Ld\, N \simeq 1600\, bit$ .
   The respective value of information i = Ld N will further be reduced considerably if we introduce the facts that each point has 4-liberties for each move. Thereby we must properly write $i = {}^4 \log N$ for expressing information. This kind of argumentation and estimations may still be considerably refined with more precision and a lot more effort.

**III.d) Uncertain rules:**

Despite all calculations and refinement we can perceive very soon some principal difficulties and limits of understanding and quantitative definition of positions and plurality of moves. In existing rules as they are, we do not see - up to now - a logic link between the use of liberties and counting of points. ( This might be seen with valueassessment of Seki-positions, of suicide-rules, of multiple-Ko and „entire-Ko".)

It might be a fundamental error to assume the aim of the game ( Spielziel ) of Go to be the amximisation of only-and-merely the points. There is a strong feeling that looking at the way the game is played rather a maximization of liberties is more suitable. Further- more the rule of capturing (Schlagrecht ) in a sense of elementary senté merits much more of special consideration.

How should this be regarded in respects of final count? It is only by principle of maximizing according to strictly logical rules that it will be possible to separate the lot of nonsense moves (negative playing ) from playing according to true rules. In this sense we should understand information content and redundance. (By this meaning we only may distinguish them from pseudo forms. )

We continue now with some special considerations:

## IV. Special analysis concerning values of the Go-board

### a.) Extrema:

For capturing one single stone it is evident that an extreme effort of his opponent is necessary to do this. By this effect one single stone is rather safe. For small groups of stones (lets say for example 4 stones ) the same principle is true. Quite differently we must look at groups bigger than 12 stones. There the opponent by geometrical effect needs less stones in the group. The principle is for a collective „n“ : for the area it is ~ $n^2$ . That is more than the circumference with $\approx n$ i.e. the number of stones to surround the area. For exactly n = 12, this is a case of instability where the inner group would be suffering from geometrical drawback, for just in this case the inner group may get absolute life.

Similar conditions are in the corner for n = 6. The principle we are studying here means: that with around 12 points attack and defence are very well balanced; (Minimum).

### IV.b) The forming of value:

The forming of value at the side (edge ) of the board:

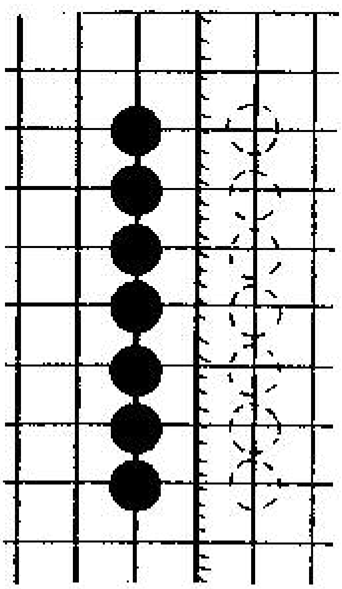


*Fig. 7*

By only 3 liberties and the rules of Go the side of the board has the property as we may call it the „mirror effect“. That means as Fig.7 shows, for example on the second line, there appears to each stone another stone in the same distance to the edge behind it like in a mirror. thereby the points on the first line develop to be secure territory or value – for the encircling stones. Or one stone develops near the side (here the 2nd line ) with its one liberty towards the side the definite property to earn (here one ) points. This consideration is valid independently of the property of life! Whereas on a onboard cannot, -by principle-, liberties be assessed by earning points, because there is no vicinity of the edge of the board that effects directly a „bonus of points“.

Thus we have to reconsider information content and the winning points by this aspect of„mirror effect“.

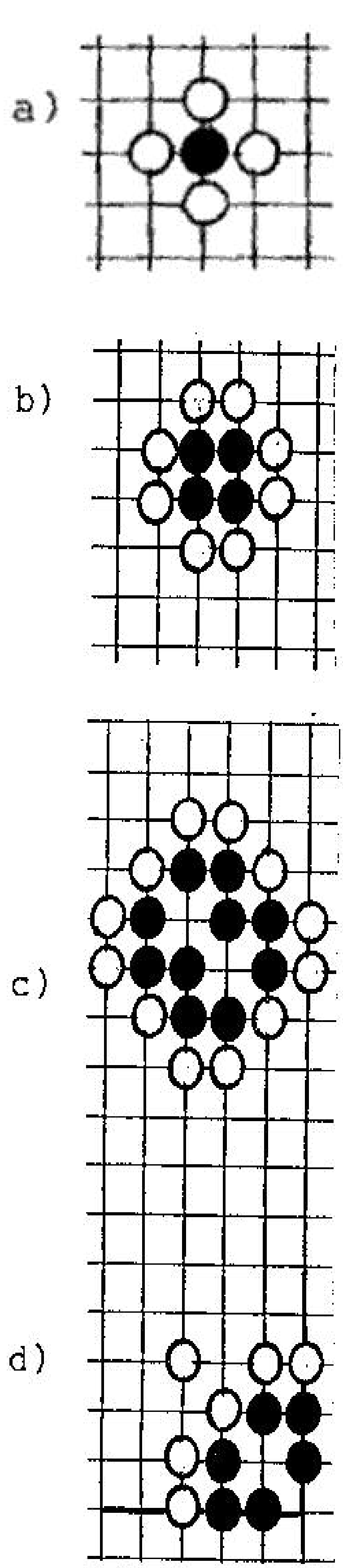



*Fig. 6*

### IV.c) Value and... sets; multitudes...; (Mengen)

Of course it is of high interest to have estimations on information content. But by calculating of positions we still have to solve the problems what does this mean with respect to points of the result. We have to solve the logic-causal relation between the use of liberties and their coordination to 4- liberties, respectively 1-point. One thing for sure we do know on the ooboard, an evaluation of points is not possible as all points can live and must not be killed. Under certain circumstances they even can make territory. But the game can never be finished. Such conditions are principally different on a finite board (with edges). Here not optimal sequences of moves lead to a loss of points.

### IV.d) Studying small boards

How and with which conditions special strategies are to be considered optimal is still rather uncertain from the point of view of strict logic. ...( Of course, however, there are already known a lot of optimal strategies such as: Joseki, Opening, Tesuji, etc. however they can not yet be grouped in categories of strict logic Axioms.)

It is dangerous to hope for miracles in this field. Up to now a mathematician, programmer is considered a hopeless flop and failure or even incapable if he does not solve all the problems of Go including all problems of play in one treatise. To avoid such misconceptions and to have systematic progress we recommend to analyse problems of interest at first on small boards, as for example has been done by Thorp and Walden, in order to gradually gain more understanding of the problem on bigger boards.

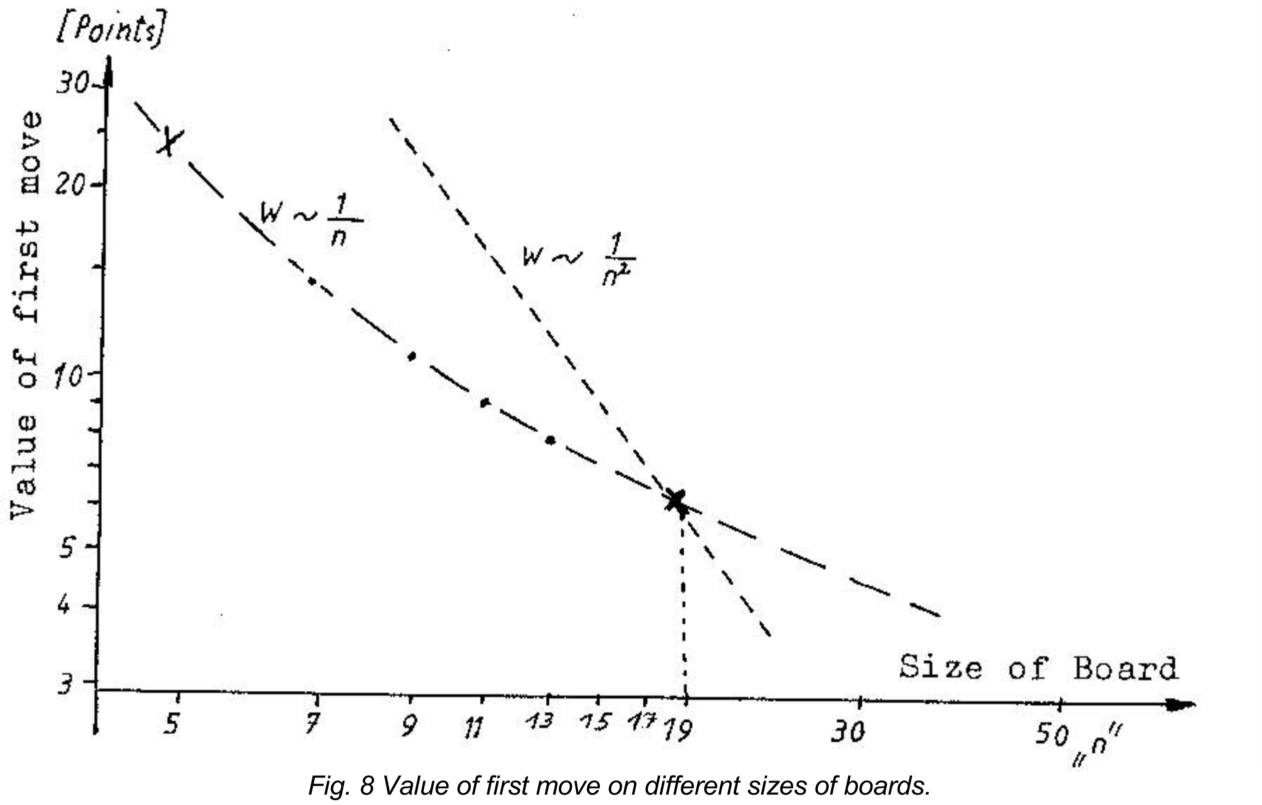



*Fig. 8 Value of first move on different sizes of boards.*

In Fig.8 we have as an example analysed the value of the first move as a function of the size of the board: $\mathrm{W}=f(n) \simeq \frac{1}{n}$ ; For a comparison the function of area $W \sim \frac{1}{n}2$ has been indicated by a dotted line. But it does not fit the problem. Here are more measurements desperately needed.

## V. <u>Conclusion</u>

We can summarise that for the game of Go the value assessment by points does fit to a meaning of information content. So that logical analysis should start with research on in-

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formation. After we did this in the second chapter, calculation of special positions is not yet possible. there are even fundamental difficult problems still to overcome.
Despite this we receive from estimations some important clues about the existence of redundace in Go. But the exact calculations are much more difficult. For this difficulty there exists more reasons from axiomatically uncertainty of the rules for the game.
Two important factors that influence value assessment with Go prove to be the function of the edge of the board and the characteristic property of 12-stones to be an extrema.
Furthermore we suggest to have much more studies on small boards. The value of the first move on smaller boards is given as an example for this.

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